Note that choice E) NOTA stands for "None of the above" answers is correct

1. Let the function $f(x)$ be continuous on $[a, b]$ and differentiable on (a, b) have the property that $f(a) = f(b)$. There exists a value c such that $a < c < b$ that satisfies Rolle's theorem. Find $f'(c)$.

A) 0   B) $\frac{b-a}{2}$ C) $\frac{f(b)-f(a)}{2}$ D) $\frac{a+b}{2}$ E) NOTA

2. Let $f(x) = x^2 + 3x + 4$, $g(x) = 3x^2$, and $h(x) = f(g(x))$. Find $h'(2)$.

A) 27   B) 84   C) 324   D) 588   E) NOTA

3. Evaluate

$$\int_{17}^{19} ((x + 1)^2 + (x + 1))dx$$

A) $\frac{2054}{3}$ B) $\frac{2282}{3}$ C) $\frac{2522}{3}$ D) $\frac{2169}{2}$ E) NOTA

4. Find

$$\lim_{x \to \infty} \frac{3x^5 - 7x^4 + 3x^3 - x^7 + 2}{9x^5 - 4x^2 + x - 1}$$

A) $-\frac{7}{9}$ B) $\frac{1}{3}$ C) $\frac{7}{9}$ D) $\frac{3}{4}$ E) NOTA

5. The base of a solid R is bounded by the x-axis, $x = 0$, $x = 5$, and the parabola, $y = x^2$. Solid R is comprised of square cross-sections perpendicular to the x-axis such that their sides lie on the bounded region defined above, what is the volume of R?

A) 125   B) 625   C) 3125   D) 15625   E) NOTA

6. Approximate $\sqrt{45}$ given that $\sqrt{49} = 7$ using differentials.

A) $\frac{45}{7}$ B) $\frac{47}{7}$ C) 7   D) $\frac{51}{7}$ E) NOTA

7. Let s_n denote an infinite sequence such that

$$s_n = \left(\frac{(n+1)^{4036}}{2018^n}\right)\left(\frac{2018^{n+1}}{(n+2)^{4036}}\right)$$

Find the value to which s_n converges.

A) $\frac{1}{2}$ B) $\frac{1}{2018}$ C) 2018   D) Divergent   E) NOTA

8. Evaluate

$$\int_0^{\frac{\pi}{2}} x^2 \sin x \, dx$$

A) $-\pi - 2$ B) $2 - \pi$ C) $\pi + 2$ D) $\pi - 2$ E) NOTA

9. Find $g'(x)$ if

$$g(x) = \int_{3x^2}^{4x^3} \sin t^2 \, dt$$

A) $12x^2 \sin x^2 - 6x \sin x^2$ B) $12x^2 \sin 9x^4 - 6x \sin 16x^6$

C) $12x^2 \sin 16x^6 - 6x \sin 9x^4$ D) $6x \sin 16x^6 - 12x^2 \sin 9x^4$ E) NOTA

10. How many of the following are convergent?

I. $\sum_{j=0}^{\infty} \frac{3^j}{4^j + j^2}$ II. $\sum_{k=0}^{\infty} \frac{2^k}{k^2}$ III. $\sum_{n=0}^{\infty}(-1)^n \sec^3(n)$ IV. $\sum_{x=0}^{\infty} \frac{(2x)!}{x^x}$

A) 1   B) 2   C) 3   D) 4   E) NOTA

11. The limit

$$\lim_{a \to 1} \frac{a - a^{6/5}}{a^{1/4} - a^{10/9}}$$

can be expressed as $\frac{n}{m}$ where n and m are relatively prime positive integers. Find $n + m$.

A) 191   B) 641   C) 209   D) 241   E) NOTA

12. Find $\frac{dy}{dx}$ if $x^3 - x\ln y = ye^x$.

A) $\frac{3x^2y - y\ln y - ye^x}{x + ye^x}$ B) $\frac{3x^2y - \ln y + y^2e^x}{x + ye^x}$ C) $\frac{3x^2y + y^2e^x}{x + xe^x}$ D) $\frac{3x^2y - y\ln y - y^2e^x}{x + ye^x}$ E) NOTA

13. Let $f(x) = x^{\sqrt{x}}$. $f'(3)$ can be expressed as $3^{\sqrt{a}-b}(\frac{1}{2}\ln(c) + d)$, where $a, b, c,$ and d are positive rational numbers. Find $2a + 4b + c - d$.

A) 10   B) 11   C) 12   D) 13   E) NOTA

14. Tammy has two birds that exist in the Cartesian plane. One bird flies along the path defined by the function $a(x) = 3x^3 - x + 1$ while the other bird flies along the path defined by $b(x) = 2x^2 + 1$. Find the area of the region bounded by the birds' flight paths.

A) $\frac{164}{81}$ B) $\frac{44}{27}$ C) $\frac{32}{81}$ D) $\frac{71}{162}$ E) NOTA

15. Find the value of

$$\int_1^{\frac{\sqrt{3}}{3}} \frac{2x^3 + 2x^2 + 3x + 1}{x^4 + x^3 + x^2 + x} dx$$

A) $\ln(3\sqrt{3} - 3) + \frac{\pi}{12}$ B) $\ln(3\sqrt{3} + 3) - \frac{\pi}{12}$

C) $\ln(\frac{\sqrt{3}}{6} + \frac{1}{6}) - \frac{\pi}{12}$ D) $\ln(\frac{\sqrt{3}}{6} - \frac{1}{6}) + \frac{\pi}{12}$ E) NOTA

16. Caleb is approximating the area under $f(x) = 3x^4 + 44x^3 - 12x^2 - 1440x + 40$ on the interval $[0,1]$ using a left hand Riemann sum with four equal subintervals. Which of the following are true?

I.   f(x) is increasing on this interval
II.   f(x) is decreasing on this interval
III. Caleb will over-approximate the value on this interval
IV.   Caleb will under-approximate the value on this interval

A) I, III   B) II, III   C) I, IV   D) II, IV   E) NOTA

17. Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{4x^n}{(n+4)!}$$

A) $(-1, 1)$ B) $(-1, 1]$ C) $(-4, 4)$ D) $(-4, 4]$ E) NOTA

18. Use a second degree Taylor polynomial centered at x = 0 to estimate the value of

$$\int_0^4 \sqrt{x^2 + 4}\, dx$$

A) 56/3   B) 28/3   C) 14/3   D) 40/3   E) NOTA

19. A stream is 20 yards wide throughout its entire length. Harry wants to build a path consisting of a road built only on land and a bridge built only over water from point A to point B which is 40 yards downstream on the other side of the stream as shown. It costs $16 and $20 to build one yard of road and bridge respectively. Find the length of bridge, in yards, that minimizes the cost of the entire path.

20 yds.

A

40 yds.

B
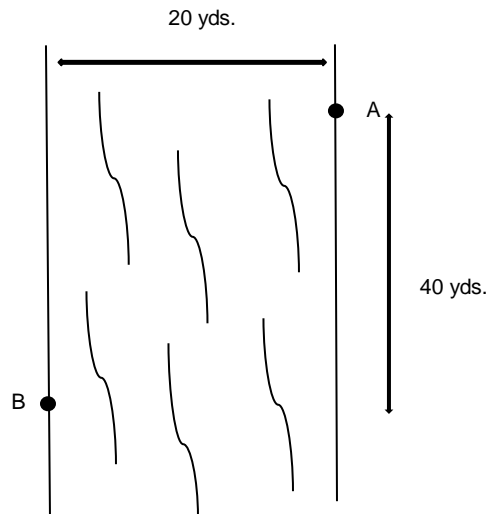
A) $\frac{80}{3}$ B) $\frac{40}{3}$ C) $\frac{100}{3}$ D) $\frac{20}{3}$ E) NOTA

20. Evaluate

$$\int \frac{1 + \cos^2(x)}{1 + \cos(2x)} dx$$

A) $\frac{1}{4}\sin(x) + \cos(x) + x + C$

B) $\sec(x)\tan(x) + C$

C) $\frac{1}{2}\ln|\sin(x) - \cos(x)| + C$

D) $\frac{1}{2}\tan(x) + \frac{1}{2}x + C$

E) NOTA

21. Find $\frac{d^2 y}{dx^2}$ at $t = 2\sqrt{2}$ if $y = \tan^{-1}(t)$ and $x = \log(t)$.

A) $\frac{-14\sqrt{2}(\ln 10)^2}{81}$

B) $20(\ln 10)$

C) $\frac{20\sqrt{2}(\ln 10)}{81}$

D) $\frac{8 (\ln 10)^2}{27}$

E) NOTA

22. The region R bounded by $x = 1$, $x = 5$, $y = 2$, and $y = \ln x + 5$ is rotated around the y-axis. Find the volume of this solid.

A) $60\pi + 25\pi \ln 5$

B) $\frac{225}{2}\pi + 25\pi \ln 5$

C) $108\pi + 25\pi \ln 5$

D) $\frac{175}{2}\pi + 25\pi \ln 5$

E) NOTA

23. The integral $\int_0^3 4e^{-x^2} dx$ can be expressed as which of the following?

A) $12\sum_{n=0}^{\infty} \frac{(-1)^n 9^n}{n!(2n+1)}$

B) $4\sum_{n=0}^{\infty} \frac{(-1)^n 9^n}{n!}$

C) $3\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!(2n+1)}$

D) $36\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!}$

E) NOTA

24. f(x) is an odd function. Let $\int_{-2}^{0} f(x)dx = -4$, $\int_{0}^{8} f(x)dx = 21$, and $\int_{0}^{-5} f(x)dx = 3$ Find $\int_{2}^{5} f(x)dx$.

A)-7   B) -1   C) 7   D)14   E) NOTA

25. Chris loves chicken noodle soup. He is filling a frustum (smaller radius on the bottom) at a rate of 12 cubic cm per second. If the frustum has a smaller radius of 10 cm, a larger radius of 25 cm, and a height of 15 cm, how fast is the radius of the surface of the soup changing, in cm per second, when the soup is 9 cm deep in the frustum?

A) $\frac{3}{2\pi}$ B) $\frac{1}{3\pi}$ C) $\frac{2}{3\pi}$ D) $\frac{1}{\pi}$ E) NOTA

26. Find the area enclosed by the curve $f(x) = x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2}$ and $g(x) = -2$ on the interval (-8, -2)

A)$\frac{1155}{8} - 2\ln 4$ B) $\frac{1251}{8} - \ln 4$ C)$\frac{1203}{8} - \ln 4$ D)$\frac{1107}{8} - 2\ln 4$ E) NOTA

27. Find the arclength of $r = \theta$ from 0 to $\sqrt{3}$ radians.

A)$\frac{1}{2}(\frac{3\sqrt{3}}{2} + \ln 2)$ B) $2 + \ln 2$ E) NOTA

C)$\frac{1}{2}(2\sqrt{3} + \ln(2 + \sqrt{3}))$ D)$\frac{3}{4}(1 - \ln(2 + \sqrt{3}))$

28. Find the value of

$$\int_0^1 \frac{\ln(x + 1)}{x}\, dx$$

A) $\frac{\pi^2}{8}$ B) $\frac{\pi^2}{8}$ C) $\frac{\pi^2}{12}$ D) $\frac{\pi^2}{24}$ E) NOTA

29. Squanchy the ferocious alien feline has a special potion that, when ingested, increases his size at a rate (in ft^3 per second) equal to four times his size in ft^3. He drinks the potion and begins growing. However, to make sure that he doesn't grow too fast, Squanchy drinks a second potion that decreases his size at a rate equal to 8 times the square of his size at the point five seconds after he drinks the first potion. How large is he in ft^3 10 seconds after ingesting the first potion if he's initially 3 ft^3?

A) $\frac{3e^{200}}{6e^{200} - 6e^{50} + 1}$ B) $\frac{3e^{100}}{6e^{100} - 6e^{20} + 1}$ C)$\frac{3e^{100}}{6e^{100} - 6e^{25} + 1}$ D)$\frac{3e^{50}}{6e^{50} - 6e^{25} + 1}$ E) NOTA

30. Prince Nebulon has assigned you a task. Evaluate

$$\int_0^1 \frac{3x + 4}{x^2 + 3x + 2}\, dx$$

A) $\ln 6$ B) $\ln \frac{9}{8}$ C) $\ln 3$ D) $\ln \frac{9}{2}$ E) NOTA